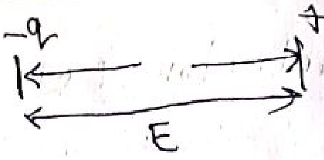
Dielectrics :- electric fields in matter, broadly Conductors & insulators. In conductors free charges are available (at least one or 2 electron per atom) are not ~~attached per~~ associated with any particular nucleus; but raam around at will. In dielectrics, all charges are attached to specific atoms or molecules they are on a tight leash (मजबूती से बंधा होना); and all they can do is move a bit within the atom or molecule. Such microscopic displacements are not as dramatic as the wholesale rearrangement of charge in a Conductor, but their cumulative effect account for the characteristic behaviour of dielectric materials. There are two principles mechanisms by which e.f. can disort the charge distribution of a dielectric atom or molecule; streching and rotating.

Induced Dipoles:- If a neutral atom is placed in an electric field $\vec{E}$, there is a positively charged core and a negatively charged electron cloud surrounding it. the nucleus is pushed in the direction of the field, and the electrons are pulled in opposite direction. If, electric field is so large, will ionize the matter then it becomes a Conductor. With less extreme fields, an equilibrium is soon established, for if the centre of the electron cloud does not coincide with the nucleus, these positive and negative charges attract one another, and this holds the atoms together.



The two opposing forces — $\vec{E}$ pulling the electrons and nucleus apart, their mutual attraction, ~~do~~ reach a balance, leaving the atom polarized, the atom now has a tiny dipole moment $\underline{p}$, which is in the same direction as $\underline{E}$. the induced dipole moment is

$$\underline{p} \propto \underline{E}$$

$$\underline{p} = \alpha \underline{E}$$

α is called atomic polarizability. α depends on the detailed structure of the atom.

Now, for case of molecules, when you apply the field along the axis of the molecule the induced dipole

$$\underline{p} = \alpha_\perp E_\perp + \alpha_{\parallel} \underline{E}_{\parallel}$$

or,

$$\left.\begin{aligned} p_x &= \alpha_{xx} E_x + \alpha_{xy} E_y + \alpha_{xz} E_z \\ p_y &= \alpha_{yx} E_x + \alpha_{yy} E_y + \alpha_{yz} E_z \\ p_z &= \alpha_{zx} E_x + \alpha_{zy} E_y + \alpha_{zz} E_z \end{aligned}\right\}$$

molecules having built-in, permanent dipole moment called polar molecules.

Electric Displacement Vector D —

For a single dipole $\underline{p}$ we have a potential

$$V(\underline{r}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\hat{\imath} \cdot \underline{p}}{\imath^2}$$

we have a dipole moment $\underline{p} = P\,d\tau'$ in each volume element $d\tau'$, so the total potential is

$$V(\underline{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\hat{\imath} \cdot \overline{P}(\underline{r}')}{\imath^2}\, d\tau' \qquad \text{——— (A)}$$

but

$$\nabla'\left(\frac{1}{\imath}\right) = \frac{\hat{\imath}}{\imath^2}$$

then,

$$V = \frac{1}{4\pi\varepsilon_0} \int_V \overline{P} \cdot \nabla'\left(\frac{1}{\imath}\right) d\tau'$$

$$= \frac{1}{4\pi\epsilon_0}\left[\int_V \underline{\nabla}'\cdot\left(\frac{\underline{P}}{\mathscr{r}}\right) d\tau' - \int_V \frac{1}{\mathscr{r}}(\nabla'\cdot\underline{P})\, d\tau'\right]$$

using div. Theorem

$$V = \frac{1}{4\pi\epsilon_0}\left[\oint_S \frac{\underline{P}}{\mathscr{r}}\cdot d\underline{a}' - \int_V \frac{1}{\mathscr{r}}(\underline{\nabla}'\cdot\underline{P})\, d\tau'\right] \quad \text{—(B)}$$

the first term looks like the potential of a surface charge $\boxed{\sigma_b = \underline{P}\cdot\hat{n}}$ ($\hat{n}$ is unit normal vector)

while second term, looks like the potential of a volume charge, $\boxed{\rho_b = -\underline{\nabla}\cdot\underline{P}}$

with these def. eqn (B) becomes

$$\boxed{V(\underline{r}) = \frac{1}{4\pi\epsilon_0}\oint_S \frac{\sigma_b}{\mathscr{r}}\cdot da' + \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho_b}{\mathscr{r}}\, d\tau'}$$

It means that the potential of a polarized object is the same as that producted by a volume charge density $\rho_b = -\underline{\nabla}\cdot\underline{P}$ plus a surface charge density $\sigma_b = \underline{P}\cdot\hat{n}$

Instead of integrating the contributions of all the infinitesimal dipoles as in eqn (A). we just find the bound charges, and then calculate the field within, the dielectric then the total charge density can be written,

$$\rho = \rho_b + \rho_f$$

we know that,

$$\underline{\nabla}\cdot\underline{E} = \frac{\rho}{\epsilon_0} \quad \text{(Gauss's law)}$$

$$\epsilon_0 \cdot \underline{\nabla}\cdot\underline{E} = \rho \qquad \because \rho = \rho_b + \rho_f$$

$$\epsilon_0 \cdot \underline{\nabla}\cdot\underline{E} = \rho_b + \rho_f$$

$$\epsilon_0 \cdot \underline{\nabla}\cdot\underline{E} = -\underline{\nabla}\cdot\underline{P} + \rho_f \quad (\because \rho_b = -\underline{\nabla}\cdot\underline{P})$$

$\underline{E}$ is the total field

then,

Combining two div. terms,

$$\nabla \cdot (\epsilon_0 \underline{E} + \underline{P}) = \rho_f \quad \text{---} \quad (i)$$

~~or $\nabla \cdot D$ Now, it is defined~~

$$\cancel{\boxed{D = \epsilon_0 E + P}}$$

where,

$$\boxed{\underline{D} = \epsilon_0 \underline{E} + \underline{P}}$$

is known as the electric displacement.

In terms of D, Gauss's law - by (i)

$$\boxed{\nabla \cdot \underline{D} = \rho_f}$$

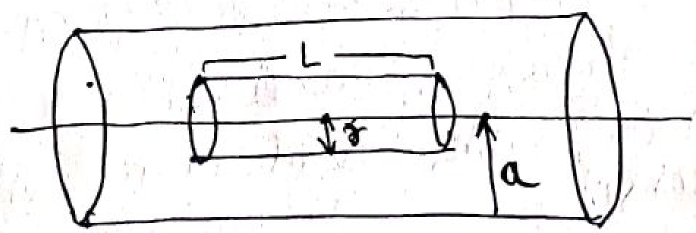
or,

$$\boxed{\oint \vec{D} \cdot d\underline{a} = Q_f}$$

Q_f denotes the total free charge enclosed in the Volume, bcoz it makes reference only to free charges in dielectric.

<u>Ex:-</u> A long straight wire, carrying uniform the charge λ, is surrounded by rubber insulation out of a radius a. Find the electric displacement.

<u>Sol:-</u>



let a cylindrical Gaussian surface, of radius r and length L, using eqn

$$\oint \underline{D} \cdot d\underline{a} = Q_f$$

since, $Q_f = \lambda L$ — L - length, λ - charge

$$\text{or, } \oint \underline{D} \cdot \hat{r}\, d\underline{a} = \lambda L$$

$$\oint \hat{r}\, da = 2\pi r L \hat{r}$$

→ gaussian surface

$\underline{D} \cdot \oint \hat{r} \cdot da = \lambda L$

$\underline{D} \cdot 2\pi r L \hat{r}\cdot\hat{r} = \lambda L$ $\quad (\hat{r}\cdot\hat{r} = 1)$

$$D = \frac{\lambda}{2\pi r}$$

$$\boxed{\underline{D} = \frac{\lambda}{2\pi r}\hat{r}}$$

if $r > a$ — (outside of ~~polarization~~ insulation there is no polarization)

$\therefore \underline{P = 0}$, so

$$\underline{D} = \frac{\lambda}{2\pi r}\hat{r} = \epsilon_0 \underline{E}$$

or,

$$\epsilon_0 \underline{E} = \frac{\lambda}{2\pi r}\hat{r}$$

$$\boxed{\underline{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}}$$ (outside the rubber)

Inside, the rubber the e.f. cannot be determined, since we do not know $\underline{P}$.

Susceptibility, Permitivity, Dielectric Constant —

since, $\underline{D} = \epsilon_0 \underline{E} + \underline{P}$ ——— (1) for many substances, the polarization is ~~proportional to the~~ field, provided $\underline{E}$ is not too strong

$$\boxed{\underline{P} = \epsilon_0 \chi_e \underline{E}}$$

those, which satisfies this type of relation are known as Linear Dielectrics.

but in (i)

$$\underline{D} = \epsilon_0 \underline{E} + \epsilon_0 \chi_e \underline{E}$$

where, χ_e is the electric susceptibility.

$$\underline{D} = \epsilon_0 (1 + \chi_e) \underline{E} \quad \text{——— (2)}$$

we know, introduce,

χ_e — chi

$$\epsilon_r = 1 + \chi_e$$

ϵ_r known relative electric permittivity or dielectric constant.

eqn (ii) can be $\vec{D} = \varepsilon_0 \cdot \varepsilon_r \cdot \vec{E}$ $Q = CV$

$\underline{D} = \epsilon \, \underline{E}$

then, ε

$\varepsilon_r = \boxed{1 + \chi_e} = \dfrac{\epsilon}{\epsilon_0}$ $\varepsilon_r = \dfrac{\varepsilon}{\varepsilon_0}$

and ϵ is permittivity and ϵ_0 is called the permittivity of free space.